Chapter 2 Sequences and Series Review

Formulae:

$$T_n = ar^{n-1} \qquad\qquad S_n = \frac{a(1-r^n)}{1-r} \qquad\qquad S_n = \frac{a-rl}{1-r}$$

$$S_\infty = \frac{a}{1-r}$$

Key points to remember:

1. The common ratio = any term divided by the previous term

2. Identify the given values for a, r, n, T_n and S_n, then select the correct formula and solve for the missing value.

3. To have a finite sum in an infinite series, $0 < |r| < 1$

4. When using sigma notation, the number of terms in any series $\sum\limits_{k=d}^{c} ar^{k-1}$ is $c - d + 1$.

5. When you see ∞ on the top of sigma notation, it is a sum to infinity.

6. For any sigma notation expression that does not appear to be geometric, expand the series first and then solve.

Review Questions

Part 1: Sequences

1. What is T_{10} if the first term in a geometric series is 16 and the common ratio is $\frac{1}{2}$?

2. Determine T_8 for the series 3, -6, 12, -24, …..

3. Determine T_{12} for the series $\frac{1}{3}$, 1, 3, 9, …..

4.   Determine the number of terms in the sequence 32, 24, 18, ………2.402709961

5.   Determine the number of terms in the sequence 5, 10, 20, 40, ………163840

6.   If $T_3 = 32$ and $T_6 = -4$, what is T_1?

7.   If $T_5 = 5$ and $T_9 = 1280$, what is T_3?

8.   Given the sequence $x,\ 8,\ x^2$, what is the value of x?

Part 2: Series

9.   Determine the sum of the series $\dfrac{1}{27} + \dfrac{1}{9} + \dfrac{1}{3} + \dots\dots + 243$.

10.   Determine the sum of the series $-16 + 8 - 4 \dots\dots + \dfrac{1}{128}$.

11.   Determine the sum of the first 6 terms of the series $15 + 5 + \dfrac{5}{3} \dots\dots$

12.   If the sum of a geometric series is 1530, determine the number of terms in the series if the first term is 6 and the common ratio is 2.

13.   If the sum of a geometric series is $\dfrac{211}{27}$, determine the value of the first term is the common ratio is $\dfrac{2}{3}$ and there are 5 terms in the series.

14.   Determine the sum of the series $10 + 5 + \dfrac{5}{2} + \dfrac{5}{4} \dots\dots$

15.   Determine the sum of the series $30 - 10 + \dfrac{10}{3} - \dfrac{10}{9} \dots\dots$

16.   For the series $(2x+1) + (2x+1)^2 + (2x+1)^3 \dots\dots$, what values of x will produce a finite sum?

17.   If the sum of an infinite series is 16 and the first term is 8, what is the value of the common ratio?

18.   If the sum of an infinite series is $\dfrac{27}{4}$ and the common ratio is $-\dfrac{1}{3}$, what is the value of the first term?

19.   Determine the sum of the series $\dfrac{1}{a-1} + 1 + (a-1) \dots\dots$ given $0 < |a-1| < 1$.

20. A ball is dropped from a height of 16 meters. After each bounce, it rises to a height of 70% of its previous height. What is the total vertical distance that it has traveled after the 8th bounce? What is the total vertical height that it has traveled after the ball comes to rest?

21. Hungtington mines near Ashcroft B.C. produced 120 tons of silver during its first year of operation. Each year production is 86% percent of its previous year. What is the total yield of silver at the end of 20 years?

Part 3: Sigma Notation

22. Solve $\displaystyle\sum_{k=1}^{9} \frac{1}{2}(3)^{k-1}$

23. Solve $\displaystyle\sum_{k=4}^{10} 2(3)^{k}$

24. $\displaystyle\sum_{k=1}^{\infty} 8\left(-\frac{1}{4}\right)^{k-1}$

25. $\displaystyle\sum_{k=1}^{\infty} 10\left(\frac{1}{2}\right)^{k}$

26. Solve $\displaystyle\sum_{k=2}^{5} k^{\log_k 2^k}$

27. $\displaystyle\sum_{k=1}^{n} 7(2)^{n-1}$

Solutions:

1. $\dfrac{1}{32}$

2. -384

3. $59\,049$

4. 10

5. 16

6. 128

7. $\dfrac{5}{16}$

8. 4

9. 364.48

10. -10.66

11. $\dfrac{1820}{81}$

12. 8

13. 3

14. 20

15. $\dfrac{45}{2}$

16. $-1 < x < 0$, $x \neq -\dfrac{1}{2}$

17. $\dfrac{1}{2}$

18. 9

19. $-\dfrac{1}{a^2 - 3a + 2}$

20. 84.52 meters, 90.67 meters

21. 815.16 tons

22. 4920.5

23. $177\,066$

24. $\dfrac{32}{5}$

25. 10

26. 60

27. $7(2)^n - 7$